Inverse participation ratio and multifractality

How to distinguish between localised and delocalised states?

Consider $I_q = \int |\psi(\vec{r})|^{2q} \, d\vec{r}$ in a finite system — inverse participation ratio (IPR)

If the state is extended, $\psi \sim \frac{1}{L^{\frac{d}{2}}}$

Then $I_q \sim L^d \cdot L^{-dq} \sim L^{-d(q-1)}$

However, if the state is localised, then $I_q \sim \xi^{-d(q-1)}$, where ξ is the localisation length

There is another possibility: fractal states: $I_q \sim L^{-d(q-1)-\Delta_q}$

$$I_q \sim \begin{cases} L^{-d(q-1)}, & \text{extended state} \\ L^{-d(q-1)-\Delta_q}, & \text{fractal state} \\ L^0, & \text{localised} \end{cases}$$

Non-linear Δ_q = multifractal states

Multifractal states occur at the metal-insulator transition